ERRATA, VOLUME 21

G. H. Hardy, *On the representation of a number as the sum of any number of squares, and in particular of five.*

Miss G. K. Stanley has applied the method of my memoir in vol. 21 of the Transactions to the seven-square problem (see Journal of the London Mathematical Society, vol 2 (1927), pp. 91–96), and has supplied me with the following list of errata. The most important is that on page 266, line 16, where a wrong convention is chosen for the meaning of $\sqrt{-k}$: the correction of this involves a considerable number of consequential corrections.

Page 263, (3. 134), read

$$``\eta(T) = \eta\left(1 - \frac{1}{T}\right) \to \frac{\pi^4}{96}" ;$$

Page 265, line 21, insert (3. 2251);

Page 266, line 12, read "$z^s = exp\{s(\log|z| + i \operatorname{am} z)\}$";

line 16, read "$\operatorname{am}(-k) = -\pi$, so that $\sqrt{-k} = -i\sqrt{k}$";

(3. 233), read

$$``\sqrt{-k}\{(-h + k\tau)i\}^{5/2} = -\sqrt{k}\{(h - k\tau)i\}^{5/2}" ;$$

line 26, read "$h<0$, $k>0$";

(3. 241), read

$$``\chi(\tau) = \frac{\pi^2}{8} + \frac{1}{2}\sum_k{}' \sum_h \frac{(-1)^h\eta}{\sqrt{k}} \frac{T_{h,k}}{\{(h - k\tau)i\}^{5/2}}$$

$$= \frac{\pi^2}{8} + \frac{\pi^2}{8}(-\tau i)^{-5/2} + \frac{1}{2}\sum{}' \frac{(-1)^h\eta}{\sqrt{k}} \frac{T_{h,k}}{\{(h - k\tau)i\}^{5/2}},$$

where $\eta = 1$ if $k>0$ and $\eta = -1$ if $k<0$, and h, k are $\cdots$";

Page 267, (3. 242), read

$$``\chi\left(-\frac{1}{\tau}\right) = \frac{\pi^2}{8} + \frac{\pi^2}{8}\left(\frac{i}{\tau}\right)^{-5/2} + \tfrac{1}{2}\sqrt{i}\sum{}' \frac{(-1)^K\eta\epsilon}{\sqrt{K}} \frac{T_{H,K}}{\{(K - H/\tau)i\}^{5/2}},$$

where $\epsilon = 1$ unless H and K are both negative, when $\epsilon = -1$, and $\eta = 1$ if $H<0$, $\eta = -1$ if $H>0$, and . . . . ";

(3. 243), read

$$\text{“}\left\{\left(K-\frac{H}{\tau}\right)i\right\}^{5/2}=\zeta\left(-\frac{1}{\tau}\right)^{5/2}\{(H-K\tau)i\}^{5/2},$$

where $\zeta=1$ unless H and K are both positive, when $\zeta=-1$, and where . . . . ”;

line 16, read

$$\text{“}-\tfrac{3}{2}\pi<\gamma=\operatorname{am}\{(H-K\tau)i\}<-\tfrac{1}{2}\pi\text{”};$$

line 17, read “$\beta+\gamma$ lies between $-\frac{3}{2}\pi$ and $\frac{1}{2}\pi$”;

line 18, read “$\alpha=\beta+\gamma$”;

line 19, read

$$\text{“}\left\{\left(K-\frac{H}{\tau}\right)i\right\}^{5/2}=\left(-\frac{1}{\tau}\right)^{5/2}\{(H-K\tau)i\}^{5/2}\text{”};$$

line 20, omit;

(3. 244), read

$$\text{“}\chi\left(-\frac{1}{\tau}\right)=\frac{\pi^2}{8}+\frac{\pi^2}{8}\left(\frac{i}{\tau}\right)^{-5/2}$$
$$+\tfrac{1}{2}\sqrt{i}\left(-\frac{1}{\tau}\right)^{-5/2}\sum{}'\frac{(-1)^K}{\sqrt{K}}\frac{\lambda T_{H,K}}{\{(H-K\tau)i\}^{5/2}},$$

where $\lambda=\epsilon\eta\zeta=1$ if $K>0$, $\lambda=-1$ if $K<0$, and where . . . . ”;

Page 268, (3. 245), insert a factor λ in each sum;

(3. 251), read

$$\text{“}\chi\left(1-\frac{1}{T}\right)=\frac{\pi^2}{8}-\frac{1}{2}\sum_H{}'\sum_K\frac{(-1)^H\eta}{\sqrt{H}}\frac{T_{H-K,H}}{\{(-K+H/T)i\}^{5/2}},$$

where $\eta=1$ if $H>0$, $\eta=-1$ if $H<0$, and H . . . .”;

line 12, read “$\operatorname{am}\{(-K+H/T)i\}$”;

last formula of footnote, read

“$\sqrt{i}(-1/\tau)^{-5/2}=-(i/\tau)^{-5/2}$”;

Page 269, line 3, read “$h>0$, $k<0$”;

(3. 255), read

$$\text{“}\chi\left(1-\frac{1}{T}\right)=\frac{\pi^2}{8}-\frac{1}{2}\sqrt{-i}\sum_H{}'\sum_K\frac{(-1)^H\epsilon\eta}{\sqrt{K}}\frac{W_{H,K}}{\{(-K+H/T)i\}^{5/2}}\text{”};$$

line 6, omit;

line 8, replace ϵ by ζ, and continue: "where $\zeta=1$ unless $H>0$, $K<0$, when $\zeta=-1$, and am T . . . . ";
line 11, read

$$``\chi\left(1-\frac{1}{T}\right)=-\frac{1}{2}\sqrt{-i}T^{5/2}\sum_H\sum_K\frac{(-1)^H\lambda}{\sqrt{K}}\frac{W_{H,K}}{\{(H-KT)i\}^{5/2}},$$

where $\lambda=1$ when $K>0$ and $\lambda=-1$ when $K<0$, and the summation is . . . . ";
(3. 256), omit $\pi^2/8$;
line 16, read

$$``\sum_{K=1,3,5,\ldots}\frac{1}{\sqrt{K}}\sum_{j=1}^{K}\sum_H\frac{(-1)^He^{(j-\frac{1}{2})^2H\pi i/K}}{\{(H-KT)i\}^{5/2}}";$$

line 20, read

$$``\sum_H\frac{(-1)^He^{(j-\frac{1}{2})^2H\pi i/K}}{\{(H-KT)i\}^{5/2}}=\sum_H\frac{(-1)^He^{H\theta_j\pi i}}{\{(H-KT)i\}^{5/2}}";$$

Page 270, line 6, for "unity" read "$\pi^2/8$";
Page 282, line 14, read "if $N\equiv3 \pmod 4$ or if $N\equiv1 \pmod 4$ and α is odd, while

$$\chi_2=1-\frac{1}{4}-\frac{1}{4.8}-\cdots-\frac{1}{4.8^{\beta-1}}-\frac{1}{4.8^{\beta}}-(-1)^{(1/4)(N-1)}\frac{1}{8^{\beta+1}}$$

if $N\equiv1 \pmod 4$ and α is even, α being equal to $2\beta+1$ when odd and to 2β when even";
line 31, add "when $N\equiv3$ or $7 \pmod 8$. If $N\equiv1$ we must replace $4.8^{\beta-1}$ by 8^β, and if $N\equiv5$ by $\frac{5}{7}\cdot8^{\beta}$";
Page 283, line 1, read "If $N\equiv3$ or 7, we have, by (7. 42)";
line 4, omit the last equality;
line 7, read

$$``8^\beta\psi\left(\frac{n}{4^\beta}\right)=2\bar{r}(n),$$

and so

$$\bar{r}(n)=\frac{80}{\pi^2}n^{3/2}\sum\left(\frac{n}{m}\right)\frac{1}{m^2}\cdot$$

When $n\equiv1$ or 5, it will be found that we obtain the same result."